Indicator Simulation for Permeability Modeling

- Discuss the Problem of Accounting for Secondary Data
- Review Conventional Techniques
- Sequential Indicator Simulation
- Examples

Indicator Coding of Data

- local hard indicator data $i(\mathbf{u}_\alpha;z)$ originating from local hard data $z(\mathbf{u}_\alpha)$:

$$i(\mathbf{u}_\alpha;z) = \mathbf{1} \text{ if } z(\mathbf{u}_\alpha) \leq z, = \mathbf{0} \text{ if not}$$

- local hard indicator data $j(\mathbf{u}_\alpha;z)$ originating from ancillary information that provides hard inequality constraints on the local value $z(\mathbf{u}_\alpha)$. If $z(\mathbf{u}_\alpha) \in (a_\alpha, b_\alpha]$, then:

$$j(\mathbf{u}_\alpha;z) = \begin{cases} 0 \text{ if } z \leq a_\alpha \\ \text{undefined (missing) if } z \in (a_\alpha, b_\alpha] \\ 1 \text{ if } z > b_\alpha \end{cases}$$

- local soft indicator data $y(\mathbf{u}_\alpha;z)$ originating from ancillary information providing prior (pre-posterior) probabilities about the value $z(\mathbf{u}_\alpha)$:

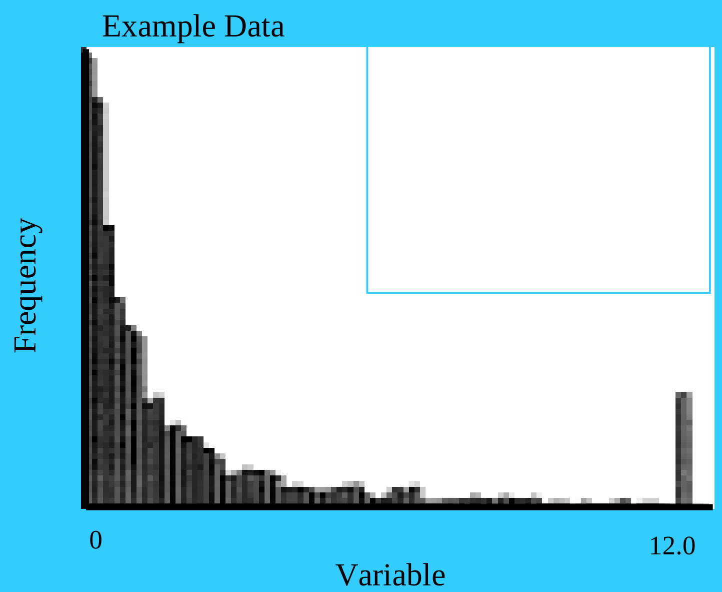
$$y(\mathbf{u}_\alpha;z) = \text{Prob}\{Z(\mathbf{u}_\alpha) \leq z \mid \text{local information}\}$$

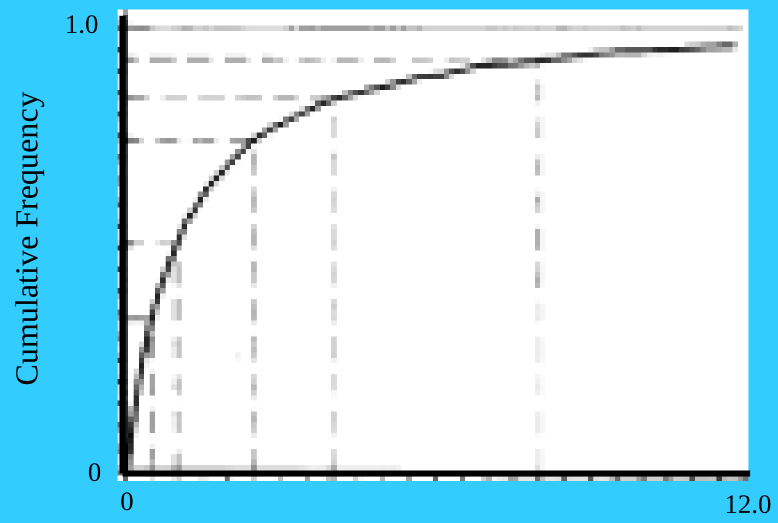
$\in [0,1]$, and $\neq F(z)$: global prior as defined hereafter

- *global* prior information common to all locations **u** within the stationary area *A*:

$$F(z) = Prob\{Z(u) \leq z\}, \ \forall u \in A$$

Indicator Kriging (IK)





- Build a local cdf conditional to surrounding data
- Can compute any of the following:
  - *E*-type estimate (local conditional mean)
  - Probability to exceed a threshold *z*
  - *z*-value of any probability
  - probability intervals
  - truncated statistics

Markov Bayes Model (1)

The IK process can be seen as a Bayesian updating of the local prior cdf into a posterior cdf using information supplied by neighboring local prior cdf's

$$[\Pr ob\{Z(u) \le z \mid (n+n')\}]^*_{IK} = \lambda_0(u)F(z) + \sum_{\alpha=1}^{n} \lambda_\alpha(u;z)i(u_\alpha;z) + \sum_{\alpha'=1}^{n'} \upsilon_{\alpha'}(u;z)y(u'_\alpha;z)$$

The $\lambda_\alpha(\mathbf{u};z)$'s are the weights attached to the *n* neighboring hard indicator data, the $\nu_\alpha(\mathbf{u};z)$'s

are the weights attached to the *n´* neighboring soft indicator data, and λ_o is the weight attributed to the global prior cdf. To ensure unbiasedness, λ_o is usually set to:

$$\lambda_0(u) = 1 - \sum_{\alpha=1}^{n} \lambda_\alpha(u;z) - \sum_{\alpha'=1}^{n'} \upsilon_{\alpha'}(u;z)$$

This can be seen as an indicator cokriging that pools information of different types: the hard *i* and *j* indicator data and the soft *y*-prior probabilities.

Markov Bayes Model (2)

The Markov-Bayes model is a model whereby the matrix of covariances is given by the following model.

$$\begin{aligned} C_{IY}(h;z) &= B(z)C_I(h;z), \quad \forall h \\ C_Y(h;z) &= B^2(z)C_I(h;z), \quad \forall h>0 \\ &= |B(z)|C_I(h;z), \quad h=0 \end{aligned}$$

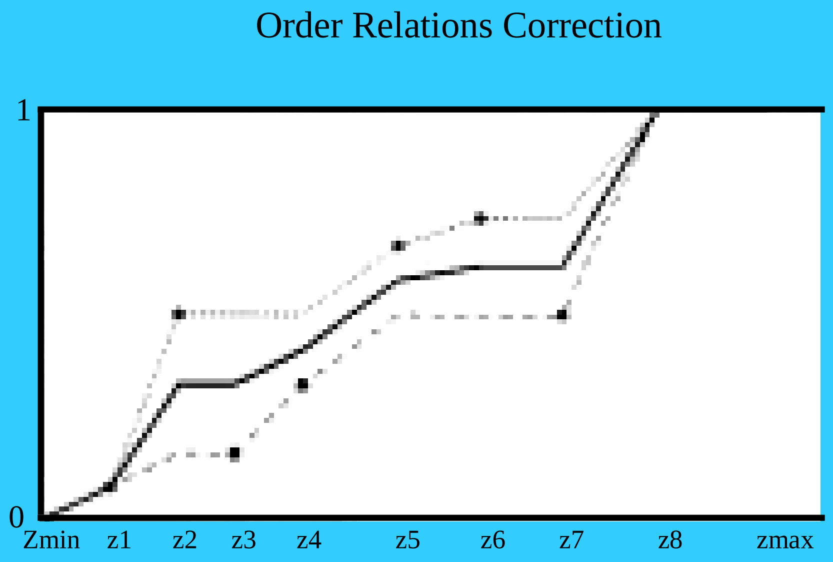
The coefficients $B(z)$ are obtained from calibration of the soft y-data to the hard z-data; more precisely:

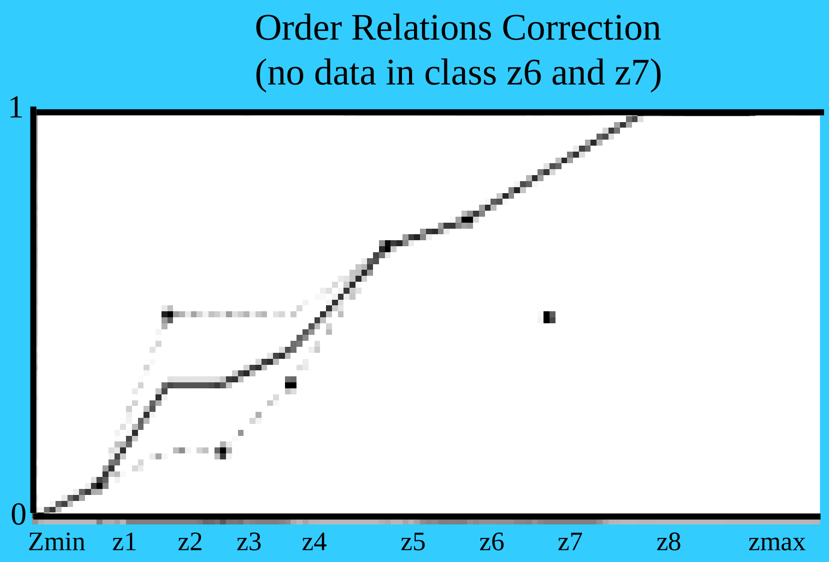
$$B(z) = m^{(1)}(z) - m^{(0)}(z) \quad \in \quad [-1,+1]$$

with:

$$\begin{aligned} m^{(1)}(z) &= E\{y(u;z) \mid I(u;z)=1\} \\ m^{(0)}(z) &= E\{y(u;z) \mid I(u;z)=0\} \end{aligned}$$

Order Relations Corrections





Parameter File (1)

```
                    Parameters for SISIM
                    ********************

START OF PARAMETERS:
1                              \ 1=continuous(cdf), 0=categorical(pdf)
5                              \ number thresholds/categories
0.5   1.0   2.5   5.0   10.0   \ thresholds / categories
0.12  0.29  0.50  0.74  0.88   \ global cdf / pdf
../data/cluster.dat            \ file with data
1   2   0   3                  \ columns for X,Y,Z, and variable
direct.ik                      \ file with soft indicator input
1   2   0   3 4 5 6 7          \ columns for X,Y,Z, and indicators
0                              \ Markov-Bayes simulation (0=no,1=yes)
0.61  0.54  0.56  0.53  0.29   \ calibration B(z) values
-1.0e21    1.0e21              \ trimming limits
0.0   30.0                     \ minimum and maximum data value
```

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Parameter File (2)

1 0.0	\ lower tail option and parameter
1 1.0	\ middle option and parameter
1 30.0	\ upper tail option and parameter
cluster.dat	\ file with tabulated values
3 0	\ columns for variable, weight
0	\ debugging level: 0,1,2,3
sisim.dbg	\ file for debugging output
sisim.out	\ file for simulation output
1	\ number of realizations
50 0.5 1.0	\ nx,xmn,xsiz
50 0.5 1.0	\ ny,ymn,ysiz
1 1.0 10.0	\ nz,zmn,zsiz
69069	\ random number seed
12	\ maximum original data for each kriging
12	\ maximum previous nodes for each kriging

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Parameter File (3)

12	\ maximum previous nodes for each kriging
1	\ maximum soft indicator nodes for kriging
1	\ assign data to nodes? (0=no,1=yes)
0 3	\ multiple grid search? (0=no,1=yes),num
0	\ maximum per octant (0=not used)
20.0 20.0 20.0	\ maximum search radii
0.0 0.0 0.0	\ angles for search ellipsoid
0 2.5	\ 0=full IK, 1=median approx. (cutoff)
0	\ 0=SK, 1=OK
1 0.15	\ One nst, nugget effect
1 0.85 0.0 0.0 0.0	\ it,cc,ang1,ang2,ang3
10.0 10.0 10.0	\ a_hmax, a_hmin, a_vert
1 0.10	\ Two nst, nugget effecT